Chapter 14
Symbolic Calculation

This chapter describes symbolic processing in Mathcad. The chapter includes the following sections:

Overview of symbolic math

Mathcad's symbolic math features and how they differ from numeric features.

Live symbolic evaluation

Using the symbolic equal sign to perform live symbolic transformations.

Using the Symbolics menu

Menu commands to perform static symbolic transformations.

Examples of symbolic calculation

Calculus, equation solving, matrix algebra, and transform examples.

Symbolic optimization

Symbolically simplifying complex expressions before numerically evaluating them.

Overview of symbolic math

Elsewhere in this *User's Guide*, you've seen Mathcad engaging in *numerical* mathematics. This means that whenever you evaluate an expression, Mathcad returns one or more *numbers,* as shown at the top of Figure 14-1. When Mathcad engages in *symbolic* mathematics, however, the result of evaluating an expression is generally another expression, as shown in the bottom of Figure 14-1.

A numerical calculation gives nothing but numbers:

$$F(x) := \sum_{k=0}^{3} \frac{3!}{k!\cdot(3-k)!}\cdot x^k \cdot 2^{3-k}$$

$$F(2) = 64$$

$$F(-5) = -27$$

But a symbolic transformation can yield insight into the underlying expression:

$$F(x) \rightarrow 8 + 12\cdot x + 6\cdot x^2 + x^3$$

Figure 14-1: A numerical and symbolic evaluation of the same expression.

There are three ways to perform a symbolic transformation on an expression.

- You can use the symbolic equal sign as described in "Live symbolic evaluation" on page 279. This method feels very much as if you're engaging in numerical math. If you need more control over the symbolic transformation, you can use *keywords* with the symbolic equal sign.
- You can use commands from the **Symbolics** menu. See "Using the Symbolics menu" on page 287.
- You can make the numerical and symbolic processors work together, the latter simplifying an expression behind the scenes so that the former can work with it more efficiently. This is discussed in "Symbolic optimization" on page 299.

Note For a computer, symbolic operations are, in general, much more difficult than the corresponding numerical operations. In fact, many complicated functions and deceptively simple-looking functions have no closed-form integrals or roots.

Live symbolic evaluation

The symbolic equal sign provides a way to extend Mathcad's live document interface beyond the numerical evaluation of expressions. You can think of it as being analogous to the equal sign "=." Unlike the equal sign, which always gives a numerical result on the right-hand side, the symbolic equal sign is capable of returning *expressions*. You can use it to symbolically evaluate expressions, variables, functions, or programs.

To use the symbolic equal sign:

- Make sure that **Automatic Calculation** on the **Math** menu has a check beside it. If it doesn't, choose it from the menu.
- Enter the expression you want to evaluate.

  $$\frac{d}{dx}\left(x^3 - 2\cdot y\cdot x\right)$$

- Click → on the Symbolic toolbar or press [**`Ctrl`**]**`.`** (the Control key followed by a period). Mathcad displays a symbolic equal sign, "→."

  $$\frac{d}{dx}\left(x^3 - 2\cdot y\cdot x\right) \rightarrow$$

- Click outside the expression. Mathcad displays a simplified version of the original expression. If an expression cannot be simplified further, Mathcad simply repeats it to the right of the symbolic equal sign.

  $$\frac{d}{dx}\left(x^3 - 2\cdot y\cdot x\right) \rightarrow 3\cdot x^2 - 2\cdot y$$

The symbolic equal sign is a live operator just like any Mathcad operator. When you make a change anywhere above or to the left of it, Mathcad updates the result. The symbolic equal sign "knows" about previously defined functions and variables and uses them wherever appropriate. You can force the symbolic equal sign to ignore prior definitions of functions and variables by defining them recursively just before you evaluate them, as shown in Figure 14-6.

Figure 14-2 shows some examples of how to use the "→."

Note The "→" applies to an entire expression. You cannot use the "→" to transform only part of an expression.

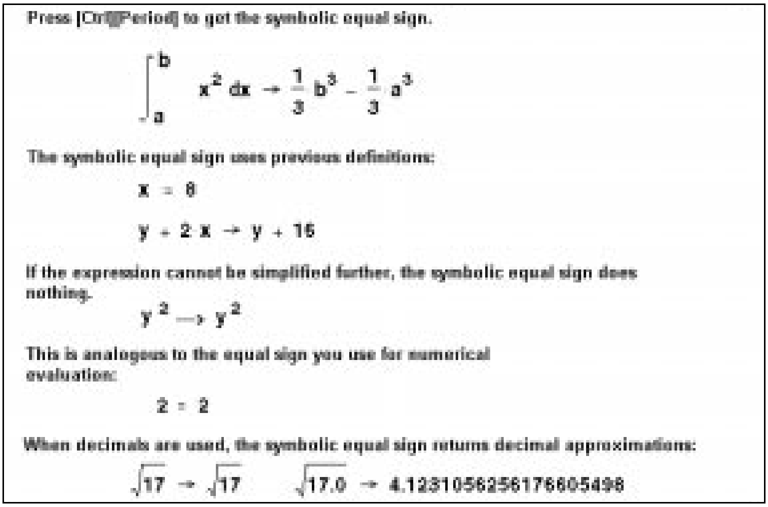


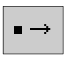
Figure 14-2: Using the symbolic equal sign.

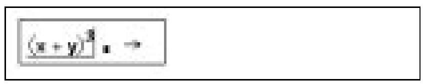
Tip Figure 14-2 also illustrates the fact that the symbolic processor treats numbers containing a decimal point differently from numbers without a decimal point. When you send numbers with decimal points to the symbolic processor, any numerical results you get back are decimal approximations to the exact answer. Otherwise, any numerical results you get back are expressed without decimal points whenever possible.

Customizing the symbolic equal sign using keywords

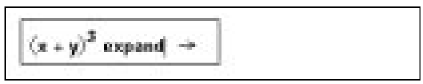
The “→” takes the left-hand side and places a simplified version of it on the right-hand side. Of course, exactly what “simplify” means is a matter of opinion. You can, to a limited extent, control how the “→” transforms the expression by using one of the *symbolic keywords*. To do so:

- Enter the expression you want to evaluate.

- Click  on the Symbolic toolbar or press [**Ctrl**] [**Shift**]**.** (press the Control and Shift keys and type a period). Mathcad displays a placeholder to the left of the symbolic equal sign, “→.”



- Click on the placeholder to the left of the symbolic equal sign and type any of the keywords from the following table. If the keyword requires any additional arguments, separate the arguments from the keyword with commas.



■ Press [**`Enter`**] to see the result.

$$(x+y)^3 \text{ expand } \rightarrow x^3 + 3\cdot x^2\cdot y + 3\cdot x\cdot y^2 + y^3$$

Tip Another way to use a keyword is to enter the expression you want to evaluate and click on a keyword button from the Symbolic toolbar. This inserts the keyword, placeholders for any additional arguments, and the symbolic equal sign, "→." Just press [**`Enter`**] to see the result.

Keyword	**Function**
`complex`	Carries out symbolic evaluation in the complex domain. Result is usually in the form $a + i \cdot b$.
`float,`*`m`*	Displays a floating point value with *m* places of precision whenever possible. If the argument *m,* an integer, is omitted, the precision is 20.
`simplify`	Simplifies an expression by performing arithmetic, canceling common factors, and using basic trigonometric and inverse function identities.
`expand,`*`expr`*	Expands all powers and products of sums in an expression except for the subexpression *expr*. The argument *expr* is optional. The entire expression is expanded if the argument *expr* is omitted. If the expression is a fraction, expands the numerator and writes the expression as a sum of fractions. Expands sines, cosines, and tangents of sums of variables or integer multiples of variables as far as possible into expressions involving only sines and cosines of single variables.
`factor,`*`expr`*	Factors an expression into a product, if the entire expression can be written as a product. Factors with respect to *expr*, a single radical or a list of radicals separated by commas. The argument *expr* is optional. Usually factors a single variable into powers of primes. Otherwise, attempts to convert the expression into a product of simpler functions. Combines a sum of fractions into a single fraction and often simplifies a complex fraction with more than one fraction bar.
`solve,`*`var`*	Solves an equation for the variable *var* or solves a system of equations for the variables in a vector *var*.
`collect,`*`var1,...,varn`*	Collects like terms with respect to the variables or subexpressions *var1* through *varn*.
`coeffs,`*`var`*	Finds coefficients of an expression when it is rewritten as a polynomial in the variable or subexpression *var*.

substitute,*var1=var2* Replaces all occurrences of a variable *var1* with an expression or variable *var2*. Press **[Ctrl]=** for the bold equal sign.

series,*var=z,m* Expands an expression in one or more variables, *var*, around the point *z*. The order of expansion is *m*. Arguments *z* and *m* are optional. By default, the expansion is taken around zero and is a polynomial of order six. By default, finds Taylor series (series in nonnegative powers of the variable) for functions that are analytic at 0 and Laurent series for functions that have a pole of finite order at 0.

convert,parfrac,*var* Converts an expression to a partial fraction expansion in *var*, the variable in the denominator of the expression on which to convert. Usually factors the denominator of the expression into linear or quadratic factors having integer coefficients and expands the expression into a sum of fractions with these factors as denominators.

fourier,*var* Evaluates the Fourier transform of an expression with respect to the variable *var*. Result is a function of ω given by:

$$\int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt$$

where $f(t)$ is the expression to be transformed.

invfourier,*var* Evaluates the inverse Fourier transform of an expression with respect to the variable *var*. Result is a function of *t* given by:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{i\omega t}d\omega$$

where $F(\omega)$ is the expression to be transformed.

laplace,*var* Evaluates the Laplace transform of an expression with respect to the variable *var*. Result is a function of *s* given by:

$$\int_{0}^{+\infty} f(t)e^{-st}dt$$

where $f(t)$ is the expression to be transformed.

invlaplace,*var* Evaluates the inverse Laplace transform of an expression with respect to the variable *var*. Result is a function of *t* given by:

$$\frac{1}{2\pi}\int_{\sigma - i\infty}^{\sigma + i\infty} F(s)e^{st}dt$$

where $F(s)$ is the expression to be transformed and all singularities of $F(s)$ are to the left of the line $\mathrm{Re}(s) = \sigma$.

ztrans,*var* Evaluates the *z*-transform of an expression with respect to the variable *var*. Result is a function of *z* given by:

$$\sum_{n=0}^{+\infty} f(n)z^{-n}$$

where $f(n)$ is the expression to be transformed.

`invztrans,`*`var`*	Evaluates the inverse z-transform of an expression with respect to the variable *var*. Result is a function of n given by a contour integral around the origin: $$\frac{1}{2\pi i}\int_C F(z)z^{n-1}dz$$ where $F(z)$ is the expression to be transformed and C is a contour enclosing all singularities of the integrand.
`assume,`*`constraint`*	Imposes constraints on one or more variables according to the expression *constraint*.

Many of the keywords take at least one additional argument, typically the name of a variable with respect to which you are performing the symbolic operation. Some of the arguments are optional. See Figure 14-3 and Figure 14-4 for examples.

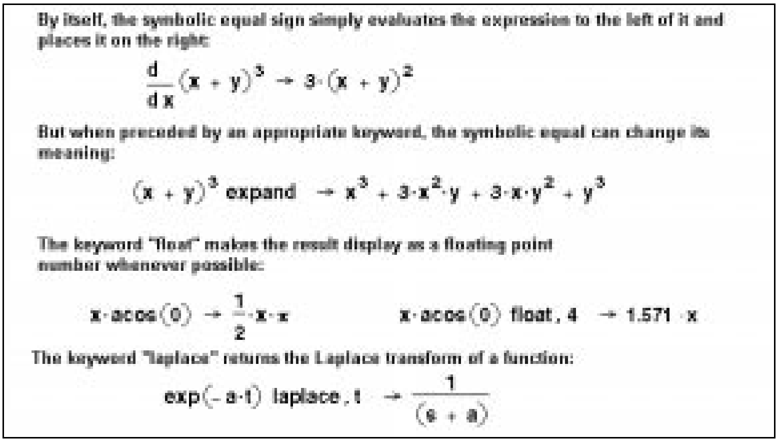


Figure 14-3: Using keywords with a symbolic evaluation sign.

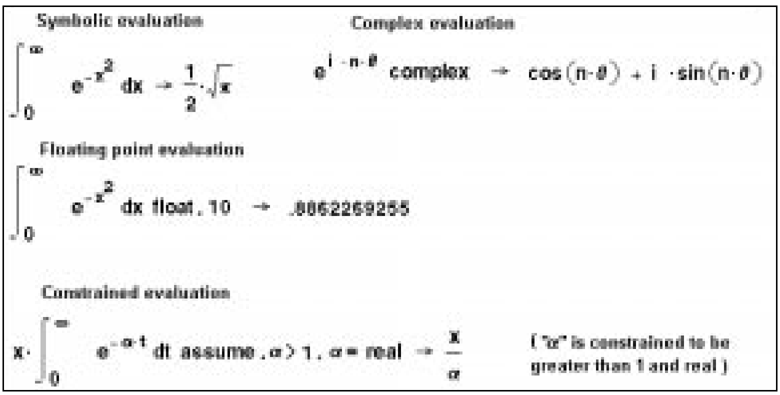


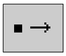
Figure 14-4: Evaluating expressions symbolically.

Note Keywords are case sensitive and must therefore be typed exactly as shown. They are not, however, font sensitive.

Keyword modifiers

Some keywords take additional modifiers that specify the kind of symbolic evaluation even further.

To use a modifier, separate it from its keyword with a comma. For example, to use the "trig" modifier with the **`simplify`** keyword on an expression:

- Enter the expression to simplify.
- Click  on the Symbolic toolbar or press [**`Ctrl`**] [**`Shift`**]**`.`** (hold down the Control and Shift keys and type a period). Mathcad displays a placeholder to the left of the symbolic equal sign, "→."
- Enter **`simplify,trig`** into the placeholder.
- Press [**`Enter`**] to see the result.

Modifiers for "assume"

`var`**`=real`**	Evaluates the expression on the assumption that the variable *var* is real.
`var`**`=RealRange(`*****`a`*****`,`*****`b`*****`)`**	Evaluates on the assumption that all the indeterminates are real and are between *a* and *b*, where *a* and *b* are real numbers or infinity ([**`Ctrl`**][**`Shift`**]**`Z`**).

Modifiers for "simplify"

`assume=real`	Simplifies on the assumption that all the indeterminates in the expression are real.
`assume=RealRange(`***`a`*****`,`*****`b`*****`)`**	Simplifies on the assumption that all the indeterminates are real and are between *a* and *b*, where *a* and *b* are real numbers or infinity ([**`Ctrl`**][**`Shift`**]**`Z`**).
`trig`	Simplifies a trigonometric expression by applying only the following identities: $\sin(x)^2 + \cos(x)^2 = 1$ $\cosh(x)^2 - \sinh(x)^2 = 1$ It does not simplify the expression by simplifying logs, powers, or radicals.

Figure 14-5 shows some examples using the **`simplify`** keyword with and without additional modifiers.

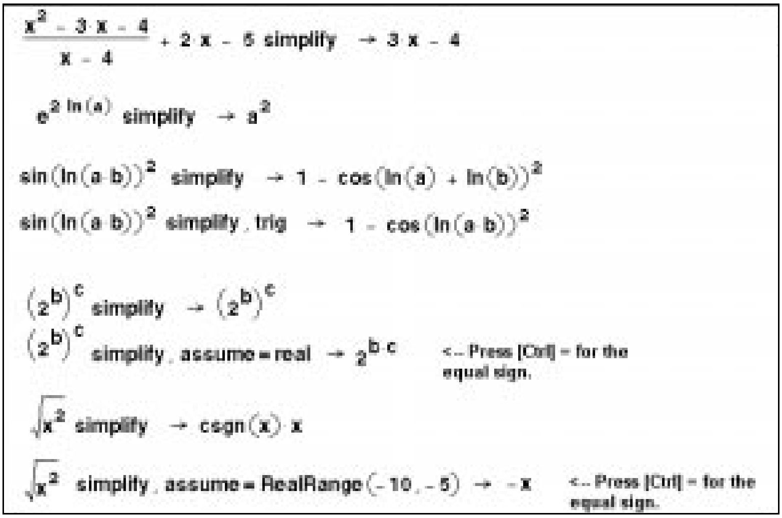


Figure 14-5: Modifiers such as “trig” allow you to control simplification.

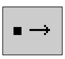
Tip Keyword modifiers can be typed or inserted from the buttons on the Modifier toolbar.

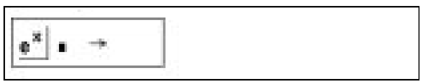
Using more than one keyword

In some cases, you may want to perform two or more types of symbolic evaluation consecutively on an expression. Mathcad allows you to apply several symbolic keywords to a single expression. There are two ways of applying multiple keywords. The method you choose depends on whether you want to see the results from each keyword or only the final result.

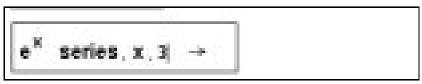
To apply several keywords and see the results from each:

- Enter the expression you want to evaluate.

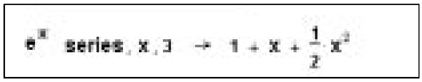
- Press  on the Symbolic toolbar or type [**`Ctrl`**] [**`Shift`**]**`.`** (hold down the Control and Shift keys and type a period). Mathcad displays a placeholder to the left of the symbolic equal sign, “→.”



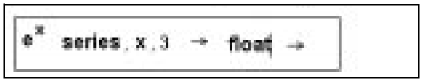
- Enter the first keyword into the placeholder to the left of the symbolic equal sign, including any comma-delimited arguments the keyword takes.



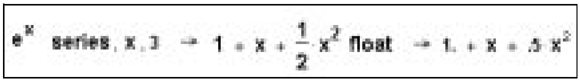
- Press [**Enter**] to see the result from the first keyword.



- Click on the result and press [**Ctrl**] [**Shift**]**.** again. The first result disappears temporarily. Enter a second keyword into the placeholder.



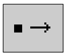
- Press [**Enter**] to see the result from the second keyword.

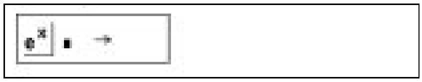


Continue applying keywords to the intermediate results in this manner.

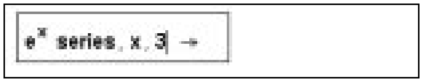
To apply several keywords and see only the final result:

- Enter the expression you want to evaluate.

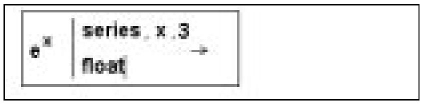
- Click  on the Symbolic toolbar or press [**Ctrl**] [**Shift**]**.** so that Mathcad displays a placeholder to the left of the symbolic equal sign, "→."



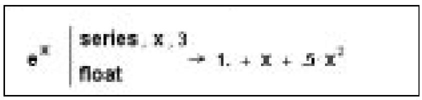
- Enter the first keyword into the placeholder, including any comma-delimited arguments it takes.



- Press [**Ctrl**] [**Shift**]**.** again and enter a second keyword into the placeholder. The second keyword is placed immediately below the first keyword.



- Continue adding keywords by pressing [**Ctrl**] [**Shift**]**.** after each one. Press [**Enter**] to see the final result.



Ignoring previous definitions

When you use the symbolic equal sign to evaluate an expression, Mathcad checks all the variables and functions making up that expression to see if they've been defined earlier in the worksheet. If Mathcad does find a definition, it uses it. Any other variables and functions are evaluated symbolically.

There are two exceptions to this. In evaluating an expression made up of previously defined variables and functions, Mathcad *ignores* prior definitions:

- When the variable has been defined recursively.
- When the variable has been defined as a range variable.

These exceptions are illustrated in Figure 14-6.

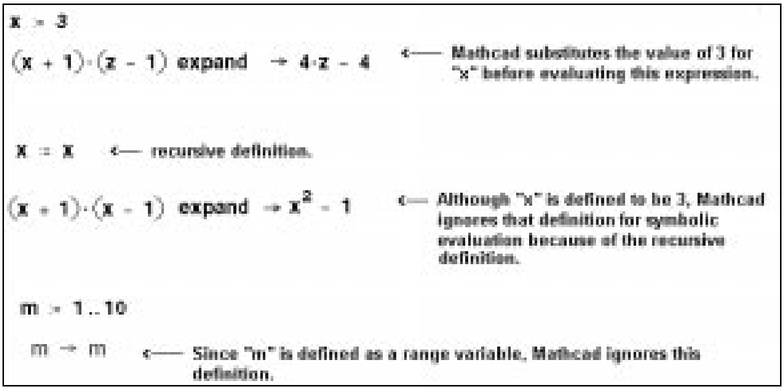


Figure 14-6: Defining a variable in terms of itself makes the symbolic processor ignore previous definitions of that variable.

Using the Symbolics menu

One advantage to using the symbolic equal sign, sometimes together with keywords and modifiers as discussed in the last section, is that it is "live," just like the numerical processing in Mathcad. That is, Mathcad checks all the variables and functions making up the expression being evaluated to see if they've been defined earlier in the worksheet. If Mathcad does find a definition, it uses it. Any other variables and functions are evaluated symbolically. Later on, whenever you make a change to the worksheet, the results automatically update. This is useful when the symbolic and numerical equations in the worksheet are tied together.

There may be times, however, when a symbolic calculation is quite separate from the rest of your worksheet and does not need to be tied to any previous definitions. In these cases, you can use commands from the **Symbolics** menu. These commands are not live: you apply them on a case by case basis to selected expressions, they do not "know" about previous definitions, and they do not automatically update.

The commands on the **Symbolics** menu perform the same manipulations as many of the keywords listed on page 280. For example, the **Symbolics** menu command **Polynomial Coefficients** evaluates an expression just as the keyword **`coeffs`** does. The only differences are that the menu command does not recognize previous definitions and does not automatically update.

The basic steps for using the **Symbolics** menu are the same for all the menu commands:

- Place whatever math expression you want to evaluate *between the two editing lines.* You can drag-select a part of the expression to place it between the editing lines.
- Choose the appropriate command from the **Symbolics** menu. Mathcad then places the evaluated expression into your document.

For example, to evaluate an expression symbolically using the **Symbolics** menu, follow these steps:

- Enter the expression you want to evaluate.

$$\frac{d}{dx}\left(x^3 - 2\cdot y\cdot x\right)$$

- Surround the expression with the editing lines.

$$\frac{d}{dx}\left(x^3 - 2\cdot y\cdot x\right)$$

- Choose **Evaluate⇒Symbolically** from the **Symbolics** menu. Mathcad places the evaluated expression into your worksheet. The location of the result in relation to the original expression depends on the Evaluation Style you've selected (see page 288).

$$3\cdot x^2 - 2\cdot y$$

Some commands on the **Symbolics** menu require that you click on or select the variable of interest rather than select the entire expression. If a menu command is unavailable, try selecting a single variable rather than an entire expression.

Tip Since the commands on the **Symbolics** menu operate only on the part of the expression currently selected by the editing lines, they are useful when you want to address parts of an expression. For example, if evaluating or simplifying the entire expression doesn't give the answer you want, try selecting a subexpression and choose a command from the **Symbolics** menu.

Long answers

Symbolic calculations can easily produce answers so long that they don't fit conveniently in your window. If you obtain a symbolic result consisting of several terms by using commands on the **Symbolics** menu, you can reformat such an answer by using Mathcad's "Addition with line break" operator (see "Operators" on page 334 in the Appendices).

Sometimes, a symbolic answer is so long that you can't conveniently display it in your worksheet. When this happens, Mathcad asks if you want the answer placed in the Clipboard. If you click "OK," Mathcad places a string representing the answer on the Clipboard. When you examine the contents of the clipboard, you'll see an answer written in a Fortran-like syntax. See on-line Help for more information on this syntax.

Displaying symbolic results

If you're using the symbolic equal sign, "→," the result of a symbolic transformation always goes to the right of the "→." However, when you use the **Symbolics** menu, you can tell Mathcad to place the symbolic results in one of the following ways:

- The symbolic result can go below the original expression.
- The symbolic result can go to the right of the original expression.
- The symbolic result can simply replace the original expression.

In addition, you can choose whether you want Mathcad to generate text describing what had to be done to get from the original expression to the symbolic result. This text goes between the original expression and the symbolic result, creating a narrative for the symbolic evaluation. These text regions are referred to as “evaluation comments.”

To control both the placement of the symbolic result and the presence of narrative text, choose **Evaluation Style** from the **Symbolics** menu to bring up the “Evaluation Style” dialog box.

Examples of symbolic calculation

Just as you can carry out a variety of numerical calculations in Mathcad, you can carry out all kinds of symbolic calculations. As a general rule, any expression involving variables, functions, and operators can be evaluated symbolically using either the symbolic equal sign or the menu commands, as described earlier in this chapter.

Tip When deciding whether to use the symbolic equal sign or menu commands from the **Symbolics** menu, remember that unlike the keyword-modified expressions, expressions modified by commands from the **Symbolics** menu do not update automatically, as described in the section “Using the Symbolics menu” on page 287.

This section describes how to symbolically evaluate definite and indefinite integrals, derivatives, and limits. It also covers how to symbolically transpose, invert, and find the determinant of a matrix. Finally, this section describes how to perform symbolic transforms and solve equations symbolically. Keep in mind that these are just a few of the calculations you can perform symbolically.

Note Functions and variables you define yourself are recognized by the symbolic processor when you use the symbolic equal sign. They are not, however, recognized when you use the **Symbolics** menu commands.Figure 14-7 shows the difference.

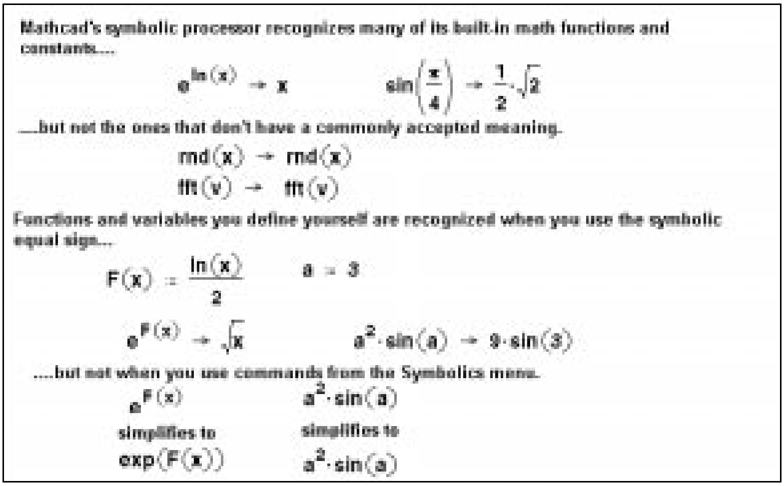


Figure 14-7: The symbolic processor recognizes certain built-in functions. Functions and variables you define yourself are only recognized when you use the symbolic equal sign.

Derivatives

To evaluate a derivative symbolically, you can use Mathcad's derivative operator and the live symbolic equal sign as shown in Figure 14-8:

- Click on the Calculus toolbar or type **?** to insert the derivative operator.

  Alternatively, click on the Calculus toolbar or type [**Ctrl**]**?** to insert the *n*th order derivative operator.

- Enter the expression you want to differentiate and the variable with respect to which you are differentiating in the placeholders.

- Click on the Symbolic toolbar or press [**Ctrl**]**.** (the Control key followed by a period). Mathcad displays a symbolic equal sign, "→."

- Press [**Enter**] to see the result.

Figure 14-9 shows you how to differentiate an expression without using the derivative operator. The **Symbolics** menu command **Variable⇒Differentiate** differentiates an expression with respect to a selected variable. For example, to differentiate $2 \cdot x^2 + y$ with respect to *x*:

- Enter the expression.
- Click on or select the *x*.
- Choose **Variable⇒Differentiate** from the **Symbolics** menu. Mathcad displays the derivative, $4 \cdot x$. Note that y is treated as a constant.

If the expression in which you've selected a variable is one element of an array, Mathcad differentiates only that array element. To differentiate an entire array, differentiate each element individually: select a variable in that element and choose **Variable⇒Differentiate** from the **Symbolics** menu.

Tip Be sure to select a variable in an expression before choosing from the **Symbolics** menu. Otherwise, the **Variable⇒Differentiate** menu command is not available.

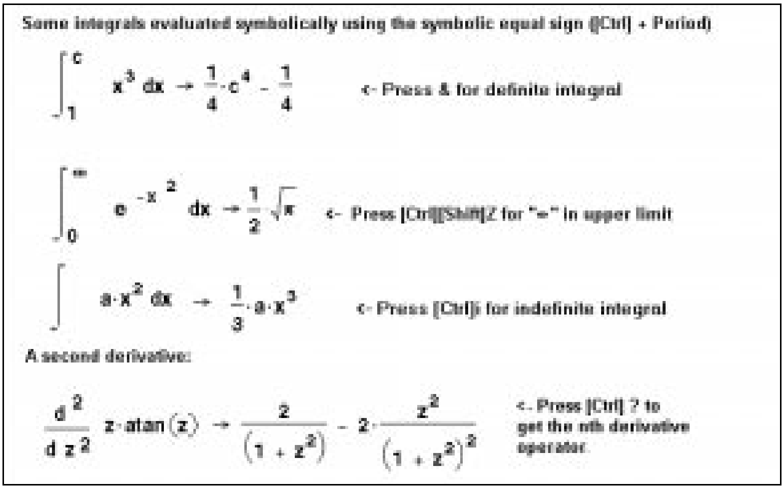


Figure 14-8: Evaluating integrals and derivatives symbolically.

Integrals

To symbolically evaluate a definite or indefinite integral:

- Click or on the Calculus toolbar to insert the definite or indefinite integral operator.
- Fill in the placeholder for the integrand and, if applicable, the placeholders for the limits of integration.
- Place the integration variable in the placeholder next to the "*d*." This can be any variable name.
- Click on the Symbolic toolbar or press [`Ctrl`]`.` (the Control key followed by a period). Mathcad displays a symbolic equal sign, "→."
- Press [`Enter`] to see the result.

See Figure 14-8 for examples of integrals evaluated symbolically.

When evaluating a definite integral, the symbolic processor attempts to find an indefinite integral of your integrand before substituting the limits you specified. If the symbolic integration succeeds and the limits of integration are integers, fractions, or exact constants like π, you get an exact value for your integral. If the symbolic processor can't find a closed form for the integral, you'll see an appropriate error message.

Another way to integrate an expression indefinitely is to enter the expression and click on the variable of integration. Then choose **Variable⇒Integrate** from the **Symbolics** menu**.** See Figure 14-9 for an example. Be sure to select a variable in an expression before choosing from the **Symbolics** menu. Otherwise, the **Variable⇒Integrate** menu command is unavailable.

Tip When you apply the **Variable⇒Integrate** command on the **Symbolics** menu, the expression you select should not usually include the integral operator. You should select only an expression to integrate. If you include the integral operator in the selected expression, you are taking a double integral.

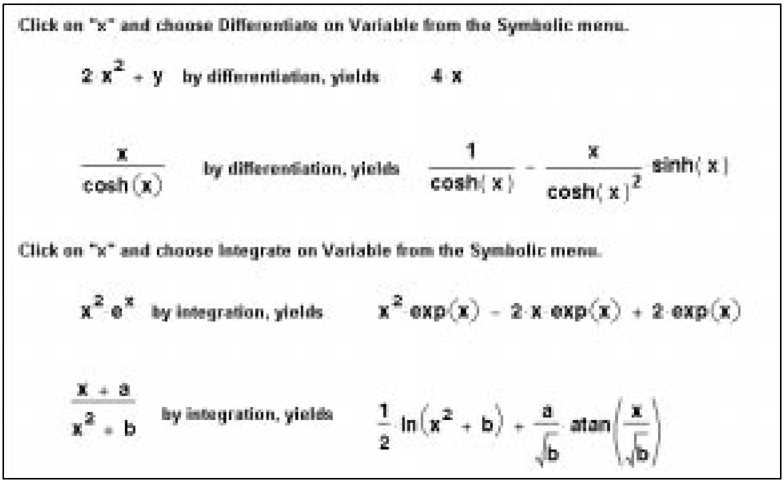
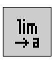
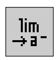
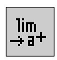


Figure 14-9: Differentiating and integrating with menu commands.

Limits

Mathcad provides three limit operators. These can only be evaluated symbolically. To use the limit operators:

- Click  on the Calculus toolbar or press [**`Ctrl`**]**L** to insert the limit operator.

  To insert the operator for a limit from the left or right, click  or  on the Calculus toolbar or press [**`Ctrl`**][**`Shift`**]**B** or [**`Ctrl`**][**`Shift`**]**A**..
- Enter the expression in the placeholder to the right of the "lim."
- Enter the limiting variable in the left-hand placeholder below the "lim."
- Enter the limiting value in the right-hand placeholder below the "lim."

■ Click on the Symbolic toolbar or press [**Ctrl**]**.** (the Control key followed by a period). Mathcad displays a symbolic equal sign, "→."

■ Press [**Enter**] to see the result.

Mathcad returns a result for the limit. If the limit does not exist, Mathcad returns an error message. Figure 14-10 shows some examples of evaluating limits.

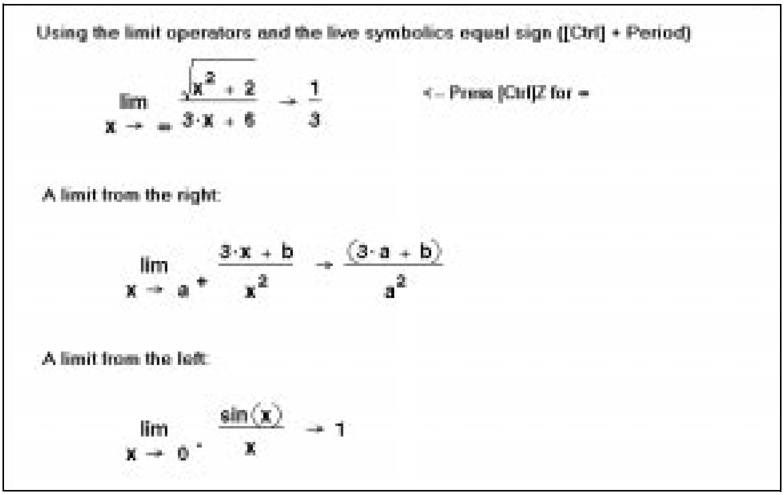


Figure 14-10: Evaluating limits.

Solving an equation for a variable

To solve an equation symbolically for a variable, use the keyword **solve**:

■ Type the equation. Make sure you click on the Evaluation toolbar or type [**Ctrl**]= to create the bold equal sign.

Note When solving for the root of an expression, there is no need to set the expression equal to zero. See Figure 14-11 for an example.

■ Click on the Symbolic toolbar or type [**Ctrl**] [**Shift**]**.** (hold down the Control and Shift keys and type a period). Mathcad displays a placeholder to the left of the symbolic equal sign, "→."

■ Type **solve** in the placeholder, followed by a comma and the variable for which to solve.

■ Press [**Enter**] to see the result.

Mathcad solves for the variable and inserts the result to the right of the "→." Note that if the variable was squared in the original equation, you may get *two* answers back when you solve. Mathcad displays these in a vector. Figure 14-11 shows an example.

Tip Another way to solve for a variable is to enter the equation, click on the variable you want to solve for in the equation, and choose **Variable⇒Solve** from the **Symbolics** menu.

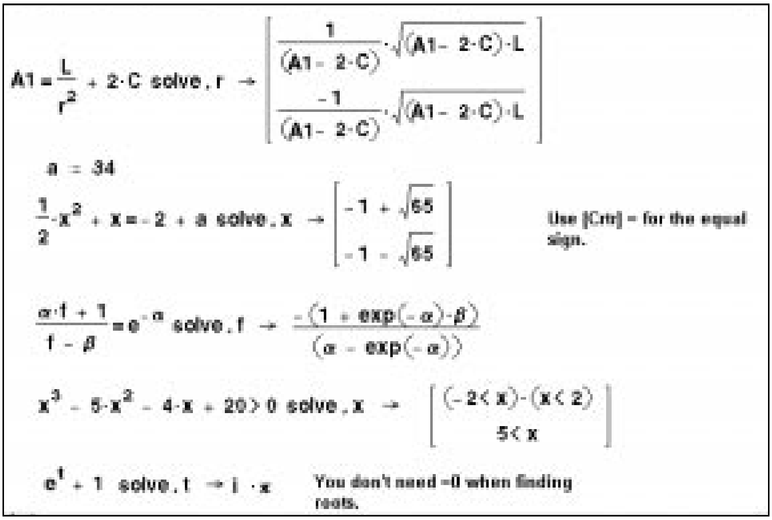


Figure 14-11: Solving equations, solving inequalities, and finding roots.

Solving a system of equations symbolically: "solve" keyword

One way to symbolically solve a system of equations is to use the same **solve** keyword used to solve one equation in one unknown. To solve a system of *n* equations for *n* unknowns:

- Press on the Matrix toolbar or type [**`Ctrl`**]**M** to insert a vector having *n* rows and 1 column.
- Fill in each placeholder of the vector with one of the *n* equations making up the system. Make sure you click on the Evaluation toolbar or type [**`Ctrl`**]= to enter the bold equal sign.
- Press on the Symbolic toolbar or type [**`Ctrl`**] [**`Shift`**]**`.`** (hold down the Control and Shift keys and type a period). Mathcad displays a placeholder to the left of the symbolic equal sign, "→."
- Type **`solve`** followed by a comma in the placeholder.

- Type [**Ctrl**]**M** or press on the Matrix toolbar to create a vector having *n* rows and 1 column. Then enter the variables you are solving for.
- Press [**Enter**] to see the result.

Mathcad displays the *n* solutions to the system of equations to the right of the symbolic equal sign. Figure 14-12 shows an example.

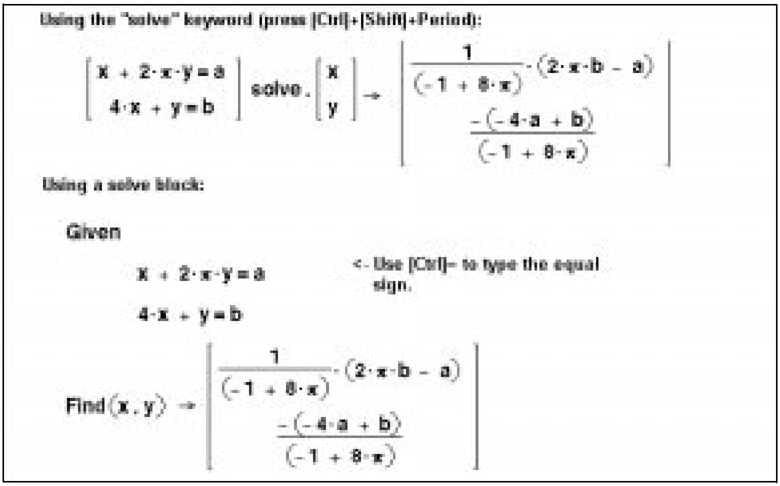


Figure 14-12: Solving a system of equations symbolically.

Solving a system of equations symbolically: solve block

Another way to solve a system of equations symbolically is to use a solve block, similar to the numerical solve blocks described in “Solving and optimization functions” on page 187:

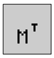
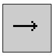
- Type the word *Given*. This tells Mathcad that what follows is a system of equations. You can type *Given* in any combination of upper- and lowercase letters and in any font. Just be sure you don’t type it while in a text region.
- Now enter the equations in any order below the word *Given*. Make sure that for every equation you click **=** on the Evaluation toolbar or type [**Ctrl**]**=** to insert the bold equal sign for each equation.
- Enter the *Find* function with arguments appropriate for your system of equations. This function is described in “Linear/nonlinear system solving and optimization” on page 189.
- Click → on the Symbolic toolbar or press [**Ctrl**]**.** (the Control key followed by a period). Mathcad displays the symbolic equal sign.
- Click outside the *Find* function or press [**Enter**].

Mathcad displays the solutions to the system of equations to the right of the symbolic equal sign. Figure 14-12 shows an example.

Most of the guidelines for solve blocks described in “Linear/nonlinear system solving and optimization” on page 189 apply to the symbolic solution of systems of equations. The main difference is that when you solve equations symbolically, you do not enter guess values for the solutions.

Symbolic matrix manipulation

You can use Mathcad to find the symbolic transpose, inverse, or determinant of a matrix using a built-in operator and the symbolic equal sign. To find the transpose of a matrix, for example:

- Place the entire matrix between the two editing lines by clicking [**`Space`**] one or more times.
- Click  on the Matrix toolbar to insert the matrix transpose operator.
- Click  on the Symbolic toolbar or press [**`Ctrl`**]**`.`** (the Control key followed by a period). Mathcad displays the symbolic equal sign, “→.”
- Press [**`Enter`**] to see the result.

Mathcad returns the result to the right of the “→.” Figure 14-13 shows some examples.

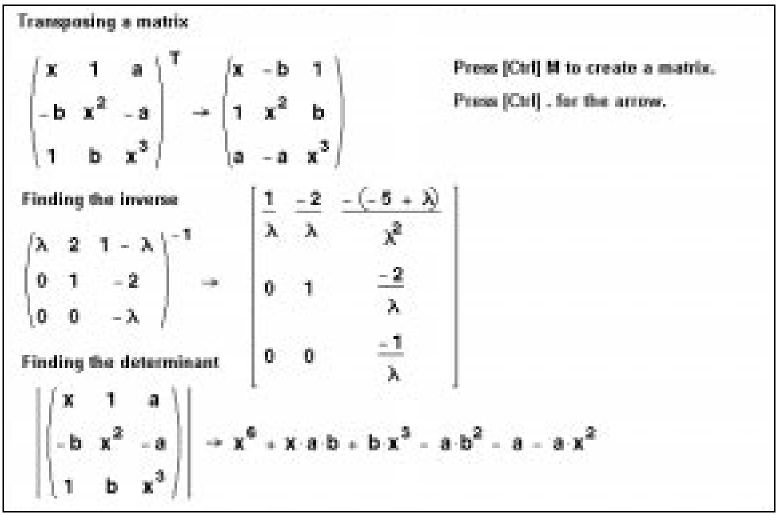


Figure 14-13: Symbolic matrix operations.

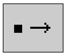
Another way to find the transpose, inverse, or determinant of a matrix is to use the **Matrix** commands on the **Symbolics** menu. For example, to find the transpose of a matrix:

- Place the entire matrix between the two editing lines by pressing [**`Space`**] one or more times.
- Choose **Matrix⇒Transpose** from the **Symbolics** menu.

Unlike matrices evaluated with the symbolic equal sign, matrices modified by commands from the **Symbolics** menu do not update automatically, as described in the section “Using the Symbolics menu” on page 287.

Transformations

You can use symbolic keywords to evaluate the Fourier, Laplace, or *z*- transform of a expression and to evaluate the inverse transform. For example, to evaluate the Fourier transform of an expression:

- Enter the expression to be transformed.
- Click  on the Symbolic toolbar or type [**`Ctrl`**] [**`Shift`**]**`.`** (hold down the Control and Shift keys and type a period). Mathcad displays a placeholder to the left of the symbolic equal sign, “→.”
- Type **`fourier`** in the placeholder, followed by a comma and the name of the transform variable.
- Press [**`Enter`**] to see the result.

Note Mathcad returns a function in a variable commonly used for the transform you perform. If the expression you are transforming already contains this variable, Mathcad avoids ambiguity by returning a function of a double variable. For example, Mathcad returns a function in the variable ω when you perform a Fourier transform. If the expression you are transforming already contains an ω, Mathcad returns a function of the variable ωω instead.

The Fourier transform result is a function of ω given by:

$$\int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt$$

Use the keyword **`invfourier`** to return the inverse Fourier transform as a function given by:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{i\omega t}d\omega$$

where $f(t)$ and $F(\omega)$ are the expressions to be transformed.

Use the keywords **`laplace`**, **`invlaplace`**, **`ztrans`**, and **`invztrans`** to perform a Laplace or *z*- transform or their inverses.

The Laplace transform result is a function of s given by:

$$\int_{0}^{+\infty} f(t)e^{-st}dt$$

Its inverse is given by:

$$\frac{1}{2\pi}\int_{\sigma - i\infty}^{\sigma + i\infty} F(s)e^{st}dt$$

where $f(t)$ and $F(s)$ are the expressions to be transformed. All singularities of $F(s)$ are to the left of the line $\mathrm{Re}(s) = \sigma$.

The z-transform result is a function of z given by:

$$\sum_{n=0}^{+\infty} f(n)z^{-n}$$

Its inverse is given by:

$$\frac{1}{2\pi i}\int_{C} F(z)z^{n-1}dz$$

where $f(n)$ and $F(z)$ are the expressions to be transformed and C is a contour enclosing all singularities of the integrand.

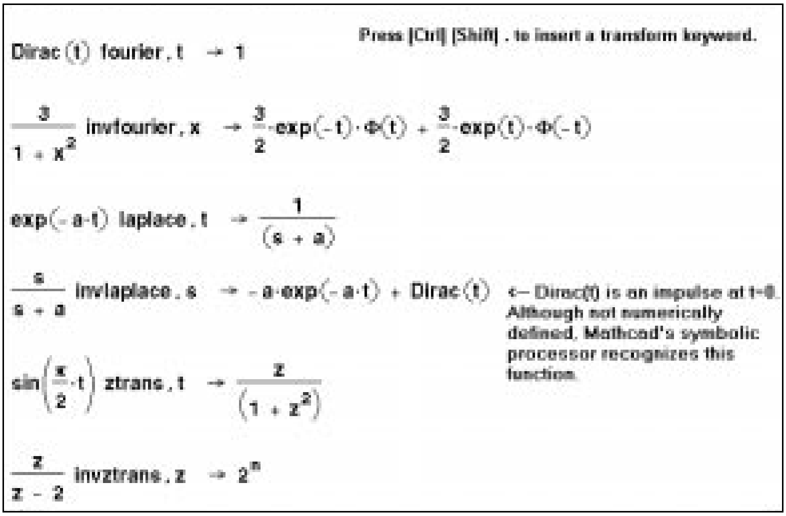


Figure 14-14: Performing symbolic transforms.

Tip You can substitute a different variable for the one Mathcad returns from a transform or its inverse by using the **substitute** keyword.

Another way to evaluate the Fourier, Laplace, or *z*- transform or their inverses on an expression is to use commands on the **Symbolics** menu. For example, to find the Laplace transform of an expression:

- Enter the expression.
- Click on the transform variable.
- Choose **Transform⇒Laplace** from the **Symbolics** menu.

Keep in mind that, unlike keyword-modified expressions, expressions modified by commands from the **Symbolics** menu do not update automatically, as described in the section "Using the Symbolics menu" on page 287.

Note Results from symbolic transformations may contain functions that are recognized by Mathcad's symbolic processor but not by its numeric processor. An example is the function *Dirac* shown in the middle of Figure 14-14. You'll find numerical definitions for this and other such functions in "Symbolic transformation functions" on page 337 in the Appendices as well as in the on-line Help.

Symbolic optimization

In general, Mathcad's symbolic and numerical processors don't communicate with one another. You can, however, make the numerical processor ask the symbolic processor for advice before starting what could be a needlessly complex calculation.

For example, if you were to evaluate an expression such as:

$$\int_0^u \int_0^v \int_0^w x^2 + y^2 + z^2 dx\, dy\, dz$$

Mathcad would undertake the laborious task of evaluating a numerical approximation of the triple integral even though one could arrive at an exact solution by first performing a few elementary calculus operations.

This happens because by itself, Mathcad's numerical processor does not know enough to simplify before plunging ahead into the calculation. Although Mathcad's symbolic processor knows all about simplifying complicated expressions, these two processors do not consult with each other. To make these two processors talk to each other, choose **Optimization** from the **Math** menu.

Once you've done this, Mathcad's live symbolic processor simplifies all expressions to the right of a ":=" *before* the numerical processor begins its calculations. This helps Mathcad's numerical processor evaluate the expression more quickly. It can also avoid any computational issues inherent in the numerical calculation.

If Mathcad finds a simpler form for the expression, it responds by doing the following:

- It marks the region with a red asterisk.
- It *internally* replaces what you've typed with a simplified form.
- The equivalent expression is evaluated instead of the expression you specified. To see this equivalent expression, double-click the red asterisk beside the region.

If Mathcad is unable to find a simpler form for the expression, it places a *blue* asterisk next to it.

In the previous example, the symbolic processor would examine the triple integral and return the equivalent, but much simpler expression:

$$\frac{1}{3}(w^3vu + wv^3u + wvu^3)$$

To see this expression in a pop-up window, click the red asterisk with the right mouse button and choose **Show Popup** from the pop-up menu (see Figure 14-15).

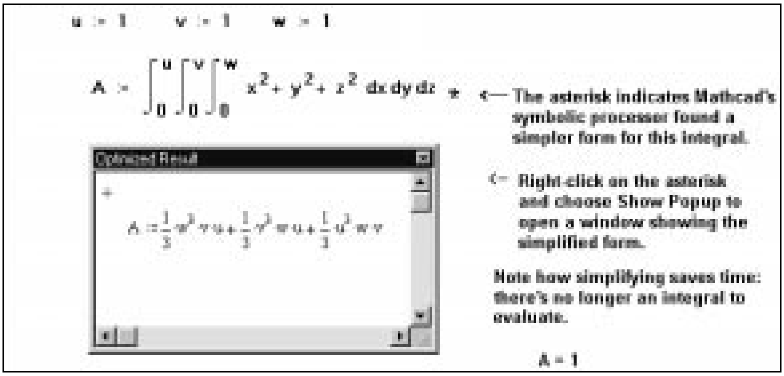


Figure 14-15: A pop-up window showing the equivalent expression that Mathcad actually evaluates.

To disable optimization for an expression, right-click it and uncheck **Optimize** on the pop-up menu. Mathcad evaluates the expression exactly as you typed it.

To disable optimization for all expressions, remove the check from **Optimization** on the **Math** menu.